# Effect of flavor mixing on the time delay of massive supernova neutrinos

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## Abstract

The neutrinos and antineutrinos of all the three flavors released from a galactic supernova will be detected in the water Cerenkov detectors. We show that even though the neutral current interaction is flavor blind, and hence neutrino flavor mixing cannot alter the total neutral current signal in the detector, it can have a non-trivial impact on the delay of massive neutrinos and alters the neutral current event rate as a function of time. We have suggested various variables of the neutral and charged current events that can be used to study this effect. In particular the ratio of charged to neutral current events can be used at early times while the ratio of the energy moments for the charged to the neutral current events can form useful diagnostic tools even at late times to study neutrino mass and mixing.

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The detection of the SN1987A neutrinos by the water Cerenkov detectors at Kamioka and IMB settled many important issues in the subject of type II supernova theory. The observation of neutrinos from any future galactic supernova event will answer the remaining questions regarding the understanding of the supernova mechanisms. A galactic supernova event will also bring in a lot of information on neutrino mass, which of late, has been an issue of much discussion. Among the various problems which demand non-zero neutrino mass are the atmospheric neutrinos anomaly [1,2], the solar neutrino problem [3,4] and the LSND experiment in Los Alamos [5]. While all the three above mentioned experiments give information on the mass squared differences, the supernova neutrinos can be used to place direct limits on the $\nu_\mu/\nu_\tau$ masses and at the same time can also constrain the neutrino mixing parameters which will be useful in the understanding of the above three experiments.

About $10^{58}$ neutrinos, in all three flavors carrying a few times $10^{53}$ ergs of energy are released in a type II supernova. These neutrinos for a galactic supernova events can be

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detected by the current water Cerenkov detectors, the Super-Kamiokande (SK) and the Sudbury Neutrino Observatory (SNO). The effect of neutrino mass can show up in the observed neutrino signal in these detectors in two ways,

- by causing delay in the time of flight measurements
- by modifying the neutrino spectra through neutrino flavor mixing

Massive neutrinos travel with speed less than the speed of light and for typical galactic supernova distances $\sim$ 10 kpc, even a small mass results in a measurable delay in the arrival time of the neutrino. Many different analyses have been performed before to give bounds on the neutrino mass ( [6] and references therein). Neutrino oscillations on the other hand convert the more energetic $\nu_\mu/\nu_\tau(\bar{\nu}_\mu/\bar{\nu}_\tau)$ into $\nu_e(\bar{\nu}_e)$ thereby hardening the resultant $\nu_e(\bar{\nu}_e)$ energy spectra and hence enhancing their signal at the detector [7–9]. In a previous work [9] we studied quantitatively the effects of neutrino flavor oscillations on the supernova neutrino spectrum and the number of charged current events at the detector using a realistic supernova model. In this work we study the neutral current signal as a function of time in the water Cerenkov detectors, for a mass range of the neutrinos where both the phenomenon of delay and flavor conversion are operative. That the time response of the event rate in the detector is modified if the neutrinos have mass alone and hence delay is a well known feature [6]. In this letter we stress the point that since neutrino flavor conversions change the energy spectra of the neutrinos, and since the time delay of the massive neutrinos is energy dependent, the time dependence of the event rate at the detector is altered appreciably in the presence of mixing. We suggest various variables which act as tools for measuring this change in the time response curve of the neutral current events and in differentiating the cases of (a) massless neutrinos (b) neutrinos with mass but no mixing and (c) neutrinos with mass as well as mixing. In particular we study the ratio of the charged current to neutral current ratio R(t), as a function of time in the SNO detector and show that the change in the value and the shape of R(t) due to flavor mixing cannot be emulated by uncertainties. We also study other variables like the normalized $n$-th energy moments of the neutral current events and the ratio of charged to the neutral current $n$-th moments as important diagnostic tools in filtering out the effects of neutrino mass and mixing.

The differential number of neutrino events at the detector for a given reaction process is

$$\frac{d^2S}{dEdt} = \sum_i \frac{n}{4\pi D^2} N_{\nu_i}(t) f_{\nu_i}(E)\sigma(E)\epsilon(E) \tag{1}$$

where $i$ runs over the neutrino species concerned, $N_{\nu_i}(t) = L_{\nu_i}(t)/\langle E_{\nu_i}(t)\rangle$, are the number of neutrinos produced at the source where $L_{\nu_i}(t)$ is the neutrino luminosity and $\langle E_{\nu_i}(t)\rangle$ is the average energy, $\sigma(E)$ is the reaction cross-section for the neutrino with the target particle, $D$ is the distance of the neutrino source from the detector (taken as 10kpc), $n$ is the number of detector particles for the reaction considered and $f_{\nu_i}(E)$ is the energy spectrum for the neutrino species involved, while $\epsilon(E)$ is the detector efficiency as a function of the neutrino energy. By integrating out the energy from eq.(1) we get the time dependence of the various reactions at the detector. To get the total numbers both integrations over energy and time has to be done.

For the neutrino luminosities and average energies, though it is best to use a numerical supernova model, but for simplicity, we will here use a profile of the neutrino luminosities

and temperatures which have general agreement with most supernova models. We take the total supernova energy radiated in neutrinos to be 3 $\times 10^{53}$ ergs. This luminosity, which is almost the same for all the neutrino species, has a fast rise over a period of 0.1 sec followed by a slow fall over several seconds in most supernova models. We use a luminosity that has a rise of 0.1 sec using one side of the Gaussian with $\sigma = 0.03$ and then an exponential decay with time constant $\tau = 3$ sec for all the flavors [6].

The average energies associated with the $\nu_e, \bar{\nu}_e$ and $\nu_\mu$ (the $\nu_\mu, \bar{\nu}_\mu, \nu_\tau$ and $\bar{\nu}_\tau$ have the same energy spectra) are 11 MeV, 16 MeV and 25 MeV respectively in most numerical models. We take these average energies and consider them to be constant in time. We have also checked our calculations with time dependent average energies and estimated its effect. The neutrino spectrum is taken to be a pure Fermi-Dirac distribution characterized by the neutrino temperature alone.

We will here be concerned with two important water Cerenkov detectors, the SK and the SNO. The column 1 of Table 1 lists all the important reactions in SK and SNO. In column 2 of Table 1 we report the calculated number of expected events for the various reactions in SNO, when neutrinos are assumed to be massless. The corresponding values for SK can be obtained by scaling the number of events in $H_2O$ to its fiducial volume of 32 kton. The detector efficiency is taken to be 1 and the energy threshold is taken to be 5 MeV for both SK and SNO [6]. For the cross-section of the $(\nu_e - d), (\bar{\nu}_e - d), (\nu_i - d)$ and $(\bar{\nu}_e - p)$ reactions we refer to [10]. The cross-section of the $(\nu_e(\bar{\nu}_e) - e^-)$ and $(\nu_i - e^-)$ scattering has been taken from [12] while the neutral current $(\nu_i -^{16} O)$ scattering cross-section is taken from [6]. For the $^{16}O(\nu_e - e^-)^{16}F$ and $^{16}(\bar{\nu}_e, e^+)^{16}N$ reactions we refer to [11] and use it's cross-sections for the detector with perfect efficiency. The expected number of events that we get agree quite well with the one reported in [9], where the results of a numerical supernova model was used.

If the neutrinos are massless then the time response of their signal at the detector reflect just the time dependence of their luminosity function at the source, which is the same for all the three flavors and hence the same for the charged current and neutral current reactions. If neutrinos have mass $\sim eV$ then they pick up a measurable delay during their course of flight from the supernova to the earth. For a neutrino of mass m (in eV) and energy E (in MeV), the delay (in sec) in traveling a distance D (in 10 kpc) is

$$\Delta t(E) = 0.515(m/E)^2 D \tag{2}$$

where we have neglected all the small higher order terms. The time response curve then has contributions from both the luminosity and the mass. We will now consider a scheme of neutrino masses such that $\Delta m_{12}^2 \sim 10^{-6} eV^2$ consistent with the solar neutrino problem [13] and $\Delta m_{13}^2 \approx \Delta m_{23}^2 \sim 1 - 10^4 eV^2$. The neutrino mass model considered here is one of several, given for the purpose of illustration only. In this scheme the atmospheric neutrino anomaly will have to be explained by the $\nu_\mu - \nu_{sterile}$ oscillation mode [2,14]. The mass range for the neutrinos as the hot component of hot plus cold dark matter scenario in cosmology is a few $eV$ only [15], which will conflict with the higher values in the range of $m_{\nu_3} = 1 - 100 eV$ that we consider here if $\nu_3$ is stable. Hence, we assume that the $\nu_3$ state is unstable but with a large enough life time so that it is does not conflict with the observations of SN 1987A [16] (even though SN1987A observations did not correspond to any $\nu_\tau$ event, one can put limits on the $\nu_3/\bar{\nu}_3$ lifetime as the $\nu_e/\bar{\nu}_e$ state is a mixture of all the three mass

eigenstates) and is also consistent with Big Bang Nucleosynthesis. In fact, from the ref. [6] we know that using the time delay technique, the SK and SNO can be used to probe neutrino masses down to $50eV$ and $30eV$ respectively. Hence we have presented all our results for a particular representative value of $m_{\nu_3} = 40eV$. There have been proposals in the past for an unstable neutrino with mass $\sim 30eV$ and lifetime $\sim 10^{23} sec$ [17]. Since direct kinematical measurements give $m_{\nu_e} < 5eV$ [18], we have taken the $\nu_e$ to be massless and the charged current events experience no change. But since the $\nu_\tau(\bar{\nu}_\tau)$ pick up a detectable time delay (for the mass spectrum of the neutrinos that we consider here, the $\nu_\mu(\bar{\nu}_\mu)$ do not have measurable time delay), the expression for the neutral current events gets modified to,

$$\frac{dS_{nc}^d}{dt} = \frac{n}{4\pi D^2}\int dE\sigma(E)\{N_{\nu_e}(t)f_{\nu_e}(E) + N_{\bar{\nu}_e}(t)f_{\bar{\nu}_e}(E) + N_{\nu_\mu}(t)f_{\nu_\mu}(E) + \\ + N_{\bar{\nu}_\mu}(t)f_{\bar{\nu}_\mu}(E) + N_{\nu_\tau}(t-\Delta t(E))f_{\nu_\tau}(E) + N_{\bar{\nu}_\tau}(t-\Delta t(E))f_{\bar{\nu}_\tau}(E)\} \tag{3}$$

where $dS_{nc}^d/dt$ denotes the neutral current ($nc$) event rate with delay ($d$). Delay therefore distorts the neutral current event rate vs. time curve. By doing a $\chi^2$ analysis of this shape distortion one can put limits on the $\nu_\tau$ mass [6].

We next consider the neutrinos to have flavor mixing as well. The mixing angle $\sin^2\theta_{12}$ can be constrained from the solar neutrino data ($\sin^2\theta_{12} \sim 10^{-3}$) [13] while for $\sin^2\theta_{13}$ there is no experimental data to fall back upon, but from r-process considerations in the "hot bubble" of the supernova, one can restrict $\sin^2\theta_{13} \sim 10^{-6}$ [8]. In this scenario there will be first a matter enhanced $\nu_e - \nu_\tau$ resonance in the mantle of the supernova followed by a $\nu_e - \nu_\mu$ resonance in the envelope. The MSW mechanism in the supernova for the neutrino mass scheme that we consider here is discussed in details in ref. [8]. As the average energy of the $\nu_\mu/\nu_\tau$ is greater than the average energy of the $\nu_e$, neutrino flavor mixing modifies their energy spectrum. Hence as pointed out in [9], the $\nu_e$ flux though depleted in number, gets enriched in high energy neutrinos and since the detection cross-sections are strongly energy dependent, this results in the enhancement of the charged current signal. The total number of events in SNO, integrated over time in this scenario with complete flavor conversion ($P_{\nu_e\nu_e} = 0$) are given in column 2 of Table 1. Of course since the $\bar{\nu}_e$ do not have any conversion here, the $\bar{\nu}_e$ signal remains unaltered. Also as the neutral current reactions are flavor blind, the total neutral current signal remains unchanged. But whether the time response curve of the neutral current signal remains unchanged in presence of mixing, in addition to delay, is an interesting question.

If the neutrinos have mass as well as mixing, then the neutrinos are produced in their flavor eigenstate, but they travel in their mass eigenstate. The neutrino mass eigenstates will travel with different speeds depending on their mass and will arrive at the detector at different times. For the scenario that we are considering only $\nu_3$ and $\bar{\nu}_3$ will be delayed. Hence to take this delay in arrival time into account, the eq.(3) has to be rewritten in terms of the mass eigenstates. It can be shown that expression for the neutral current event rate in terms of the mass eigenstates is,

$$\frac{dS_{nc}^{do}}{dt} = \frac{n}{4\pi D^2}\int dE\sigma(E)\{N_{\nu_1}(t)f_{\nu_1}(E) + N_{\bar{\nu}_1}(t)f_{\bar{\nu}_1}(E) + N_{\nu_2}(t)f_{\nu_2}(E) \\ + N_{\bar{\nu}_2}(t)f_{\bar{\nu}_2}(E) + N_{\nu_3}(t-\Delta t(E))f_{\nu_3}(E) + N_{\bar{\nu}_3}(t-\Delta t(E))f_{\bar{\nu}_3}(E)\} \tag{4}$$

where $N_{\nu_i}$ is the $\nu_i$ flux at the source. If the neutrinos are produced at densities much higher than their resonance densities, all the mixings in matter are highly suppressed, and the

neutrinos are produced almost entirely in their mass eigenstates. For the three generation case that we are considering, $\nu_e \approx \nu_3$, $\nu_\mu \approx \nu_1$ and $\nu_\tau \approx \nu_2$. For the antineutrinos on the other hand, at the point of production in the supernova $\bar{\nu}_e \approx \bar{\nu}_1$, $\bar{\nu}_\mu \approx \bar{\nu}_2$ and $\bar{\nu}_\tau \approx \bar{\nu}_3$. Hence the above expression for the neutral current event rate in the presence of delay and mixing can be written as,

$$\frac{dS_{nc}^{do}}{dt} = \frac{n}{4\pi D^2}\int dE\sigma(E)\{N_{\nu_\mu}(t)f_{\nu_\mu}(E) + N_{\bar{\nu}_e}(t)f_{\bar{\nu}_e}(E) + N_{\nu_\tau}(t)f_{\nu_\tau}(E) + N_{\nu_\tau}(t)f_{\nu_\tau}(E) \\ + N_{\bar{\nu}_\mu}(t)f_{\bar{\nu}_\mu}(E) + N_{\bar{\nu}_\mu}(t)f_{\bar{\nu}_\mu}(E) + N_{\nu_e}(t-\Delta t(E))f_{\nu_e}(E) + N_{\bar{\nu}_\tau}(t-\Delta t(E))f_{\bar{\nu}_\tau}(E)\} \quad (5)$$

Note that the above expression does not depend on the neutrino conversion probability as the neutral current interaction is flavor blind.

In fig. 1 we have plotted the neutral current event rate for the reaction ($\nu_i + d \rightarrow n + p + \nu_i$, where $\nu_i$ stands for all the 6 neutrino species) as a function of time for massless neutrinos along with the cases for mass but no mixing (eq.(3)) and mass along with mixing (eq.(5)). The figure looks similar for the other neutral current reactions as well, apart from a constant normalization factor depending on the total number of events for the process concerned. The curves corresponding to the massive neutrinos have been given for $m_{\nu_\tau} = 40eV$. As expected, the shape of the neutral current event rate changes due to the delay of massive $\nu_\tau$. Since the delay given by eq.(2) depends quadratically on the neutrino mass, the distortion is more for larger masses [19]. But the noteworthy point is that the presence of mixing further distorts the rate vs. time curve. The reason for this distortion can be traced to the fact that the time delay $\propto 1/E^2$. As the energy spectrum of the neutrinos change due to flavor mixing, the resultant delay is also modified and this in turn alters the neutral current event rate as a function of time. In fact the flavor conversion in the supernova results in de-energising the $\nu_\mu/\nu_\tau$ spectrum and hence the delay given by eq.(2) should increase. As larger delay caused by larger mass results in further lowering of the neutral current event rate vs. time curve for early times, one would normally expect that the enhanced delay as a result of neutrino flavor conversion would have a similar effect. But the fig. 1 shows that during the first second, the curve corresponding to delay with mixing is higher than the one with only time delay. This at first sight seems unexpected. But then one realizes that while the flavor conversion reduces the average energy of the massive $\nu_\tau$ increasing its delay and hence depleting its signal at early times, it energizes the massless and hence undelayed $\nu_e$ beam, which is detected with full strength. Therefore, while for no mixing the $\nu_\tau$ gave the larger fraction of the signal, for the case with mixing it is the $\nu_e$ that assume the more dominant role, and so even though the $\nu_\tau$ arrive more delayed compared to the case without mixing, the delay effect is diluted due to the enhancement of the $\nu_e$ fraction and the depletion of the $\nu_\tau$ fraction of the neutral current events. We have also checked that although it may seem that the curve with delay and mixing can be simulated by another curve with delay alone but with smaller mass, the actual shape of the two curves would still be different. This difference in shape though may not be statistically significant and hence one may not be able to see the effect of mixing in the time delay of the neutrinos just by looking at the time response of the neutral current event rate in the present water Cerenkov detectors. We therefore look for various other variables which can be studied to compliment this.

One such variable which carries information about both the neutrino mass and their mixing is R(t), the ratio of charged to neutral current event rate as a function of time.

In fig. 2 we give the ratio R(t) of the total charged current to the neutral current event rate in $D_2O$ in SNO as a function of time. Plotted are the ratios (i) without mass, (ii) with only mixing, (iii) with delay but zero mixing and (iv) with delay and flavor mixing. The differences in the behavior of R(t) for the four different cases are clearly visible. For no mass R(t)=0.3 and since the time dependence of both the charged current and neutral current reaction rates are the same, their ratio is constant in time. As the presence of mixing enhances the charged current signal keeping the neutral current events unaltered, R(t) goes up to 0.61 for only mixing, remaining constant in time, again due to the same reason. With the introduction of delay the ratio becomes a function of time as the neutral current reaction now has an extra time dependence coming from the mass. At early times as the $\nu_\tau$ get delayed the neutral current event rate drops increasing R(t). These delayed $\nu_\tau$s arrive later and hence R(t) falls at large times. This feature can be seen for both the curves with and without mixing. The curve for only delay starts at R(t)=0.52 at t=0 sec and falls to about R(t)=0.26 at t=10 sec. For the delay with mixing case the corresponding values of R(t) are 0.83 and 0.51 at t=0 and 10 sec respectively. The important point is that the curves with and without mixing are clearly distinguishable and should allow one to differentiate between the two cases of only delay and delay with neutrino flavor conversion.

In order to substantiate our claim that the two scenarios of only delay and delay with mixing are distinguishable in SNO, we divide the time into bins of size 1 second. The number of events in each bin is then used to estimate the $\pm 1\sigma$ statistical error in the ratio R(t) in each bin and these are then plotted in fig. 2 for the typical time bin numbers 1, 4 and 7. From the figure we see that the two cases of delay, with and without mixing, are certainly statistically distinguishable in SNO for the first 6 seconds.

We next focus our attention on $M_n^{nc}(t)$, the neutral current $n$-th moments of the neutrino energy distributions [20] observed at the detector, defined as

$$M_n^{nc}(t) = \int \frac{d^2S}{dEdt} E^n dE \tag{6}$$

while the corresponding normalized moments are given by

$$\overline{M}_n^{nc}(t) = \frac{M_n^{nc}(t)}{M_0^{nc}(t)} \tag{7}$$

We have shown the behavior of the 1st normalized moment $\overline{M}_1^{nc}(t)$ in fig. 3 as a function of time in SNO. For massless neutrinos, the $\overline{M}_1^{nc}$ has a value 40.97, constant in time, as this is again a ratio and hence the time dependence gets canceled out as in the case of R(t). For the case where the $\nu_\tau$ is massive and hence delayed, it assumes a time dependence. Since the delay $\propto 1/E^2$ and since the neutrinos are produced at the source with an energy distribution, hence at each instant the lower energy $\nu_\tau$ will be delayed more than the higher energy $\nu_\tau$. Therefore $\overline{M}_1^{nc}(t)$, which gives the energy centroid of the neutral current event distribution in $D_2O$, starts from a low value 38.76 at t=0 sec as all the $\nu_\tau$ are delayed, rises sharply as the higher energy neutrinos arrive first and then falls slowly as the lower energy delayed $\nu_\tau$ start arriving. If the $\nu_\tau$ are allowed to mix with the $\nu_e$, then they are de-energized and the above mentioned effect is further enhanced. To make an estimate of whether SNO would be able to distinguish the three cases discussed above, we compute the $\pm 1\sigma$ statistical errors in the $1^{st}$ normalized moment for the two scenarios of delay, with and without mixing, and show them

for the $1^{st}$, $6^{th}$ and $11^{th}$ bins. We see that the errors involved are large enough to completely wash out the differences between the energy moments with and without neutrino mass and mixing. Hence the normalized energy moments fail to probe neutrino mass and mixing as at early times we don't see much difference between the different cases considered, while at late times the number of events become very small so that the error in $M_0^{nc}(t)$ becomes huge, increasing the error in $\overline{M}_1^{nc}(t)$.

The variable that can be a useful probe for differentiating the case for delay with mixing from the case for delay without mixing is the ratio of the unnormalized moment of the charged to neutral current events

$$r_n(t) = \frac{M_n^{cc}(t)}{M_n^{nc}(t)} \tag{8}$$

We present in fig. 4, for SNO, the $r_n(t)$ vs. time plots (for n=1) for the cases of (a) massless neutrinos (b) with mixing but no delay (c) with delay but no mixing and (d) with delay as well as mixing. Since this is a ratio, the supernova flux uncertainties get canceled out to a large extent and since the unnormalized moments have smaller statistical errors, this is a better variable than the normalized moments to observe the signatures of neutrino mixing. In the figure we have shown the $\pm 1\sigma$ statistical errors in $r_1(t)$ for the two cases of delay alone and delay with mixing, for the $1^{st}$, $8^{th}$ and $15^{th}$ bins in time, and the two cases are clearly distinguishable in SNO for early as well as late times. Note that $r_1(t)$ is different from the ratio R(t) as it gives information about the ratio of the energy centroids of the charged current and neutral current distributions as a function of time, while the latter gives only the ratio of the number of events as a function of time.

The advantage of using ratios is that, they are not only sensitive to the mass and mixing parameters but are also almost insensitive to the details of supernova models. Since they are a ratio they are almost independent of the luminosity and depend only on some function of the ratio of neutrino temperatures. All the calculations presented so far are for fixed neutrino temperatures. In order to show that the time dependence of the neutrino temperatures does not alter our conclusions much, we present our analysis with time dependent neutrino temperatures. We take

$$T_{\nu_e} = 0.16 \log t + 3.58, \quad T_{\bar{\nu}_e} = 1.63 \log t + 5.15, \quad T_{\nu_\mu} = 2.24 \log t + 6.93 \tag{9}$$

These forms for the neutrino temperatures follow from fits to the results of the numerical supernova model given in Totani *et al.* [21]. In fig. 5 we compare the ratio R(t) for the cases of delay and delay with mixing for the two cases of fixed temperatures and the time dependent temperatures. It is clear from the figure that that the time dependence of the neutrino temperatures does not have much effect on the time dependence of the ratio of the charged current to neutral current rates. In fact the two curves corresponding to fixed and time dependent temperatures, fall within $\pm 1\sigma$ statistical errorbars for both the cases of only delay and delay with mixing.

In conclusion, we have shown that even though neutrino flavor mixing cannot alter the total neutral current signal in the detector - the neutral current interaction being flavor blind, it can have a non-trivial impact on the delay of massive neutrinos, which alters the neutral current event rate as a function of time. The neutral current event rate though does not depend on the neutrino conversion probability. In order to study the effect of neutrino

mass and mixing we have suggested various variables. Of the different variables that we have presented here, the ratio of the charged to neutral current event rate R(t), can show the effect of mixing during the first few seconds, while the charged to neutral current ratio of the energy moments are useful diagnostic tools for all times. These variables are not just sensitive to flavor mixing and time delay, they are also insensitive to supernova model uncertainties and hence are excellent tools to study the effect of flavor mixing on the time delay of massive supernova neutrinos.

In this letter we have considered a mass spectrum for the neutrinos where only the $\nu_\tau$ have a measurable delay. The model considered is one of many, but one can easily extend the above formalism to include more general classes of neutrino models [22]. In addition to the energy moments that we have presented here, the $n$-th order moments of the arrival time of the neutrinos as a function of energy can also be analyzed to study the effect of neutrino mass and mixing, and we plan to present them in a future work [22].

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## Table 1

The expected number of neutrino events in SNO. To get the number of events in SK, one has to scale the number of events in $H_2O$ given here to its fiducial mass of 32 kton. The column A corresponds to massless neutrinos, column B to neutrinos with complete flavor conversion The $\nu_i$ here refers to all the six neutrino species.

| reactions in 1 kton $D_2O$ | A | B |
|---|---|---|
| $\nu_e + d \rightarrow p + p + e^-$ | 75 | 239 |
| $\bar{\nu}_e + d \rightarrow n + n + e^+$ | 91 | 91 |
| $\nu_i + d \rightarrow n + p + \nu_i$ | 544 | 544 |
| $\nu_e + e^- \rightarrow \nu_e + e^-$ | 4 | 6 |
| $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ | 1 | 1 |
| $\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^- \rightarrow \nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^-$ | 4 | 3 |
| $\nu_e +^{16} O \rightarrow e^- +^{16} F$ | 1 | 55 |
| $\bar{\nu}_e +^{16} O \rightarrow e^+ +^{16} N$ | 4 | 4 |
| $\nu_i +^{16} O \rightarrow \nu_i + \gamma + X$ | 21 | 21 |
| reactions in 1.4 kton $H_2O$ | | |
| $\bar{\nu}_e + p \rightarrow n + e^+$ | 357 | 357 |
| $\nu_e + e^- \rightarrow \nu_e + e^-$ | 6 | 9 |
| $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ | 2 | 2 |
| $\nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^- \rightarrow \nu_{\mu,\tau}(\bar{\nu}_{\mu,\tau}) + e^-$ | 6 | 5 |
| $\nu_e +^{16} O \rightarrow e^- +^{16} F$ | 2 | 86 |
| $\bar{\nu}_e +^{16} O \rightarrow e^+ +^{16} N$ | 6 | 6 |
| $\nu_i +^{16} O \rightarrow \nu_i + \gamma + X$ | 33 | 33 |

## Figure Captions

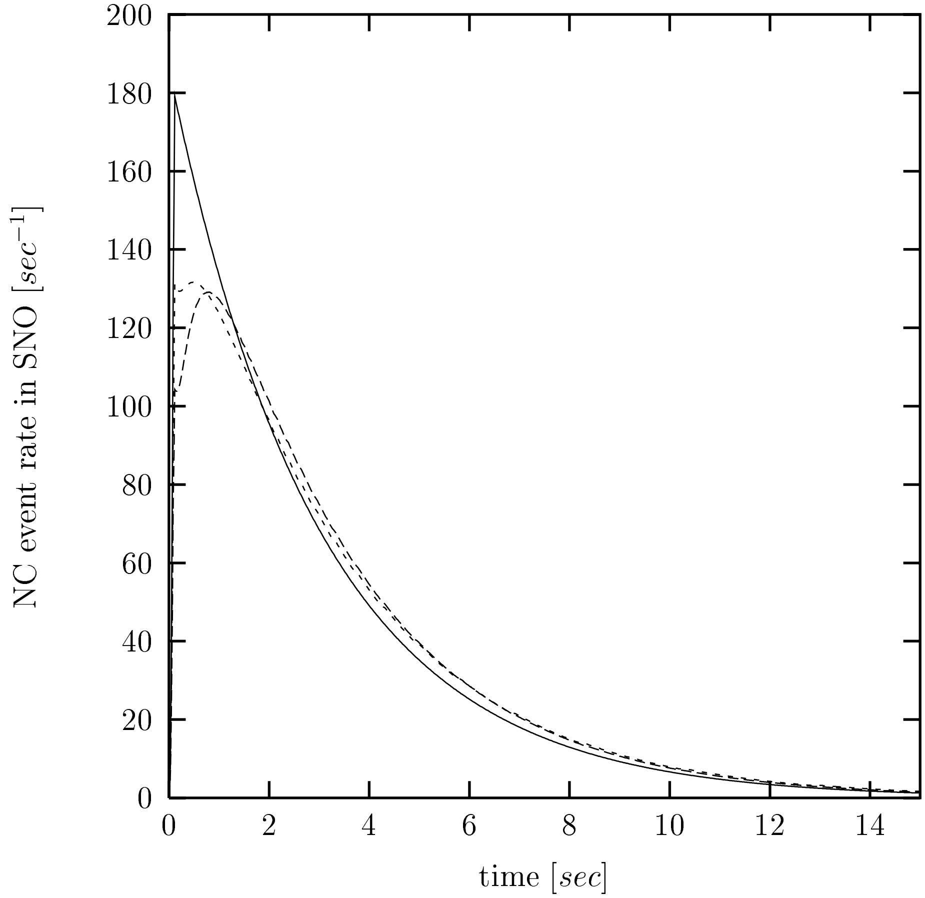
**Fig.1** The neutral current event rate as a function of time in $D_2O$ in SNO. The solid line corresponds to the case of massless neutrinos, the long dashed line to neutrinos with only mass but no mixing, while the short dashed line gives the event rate for neutrinos with mass as well as flavor mixing.

**Fig.2** The ratio R(t) of the total charged current to neutral current event rate in SNO versus time. The solid line is for massless neutrinos, the short dashed line for neutrinos with complete flavor conversion but no delay, the long dashed line for neutrinos with only delay and no flavor conversion and the dotted line is for neutrinos with both delay and complete flavor conversion. Also shown are the $\pm 1\sigma$ statistical errors for delay with and without mixing in the $1^{st}$, $4^{th}$ and the $7^{th}$ time bins.

**Fig. 3** The 1st normalized energy moment of the neutral current events in SNO $\overline{M}_1^{nc}(t)$ versus time. The solid line corresponds to the case of massless neutrinos, the long dashed line to neutrinos with only mass but no mixing, while the short dashed line gives the event rate for neutrinos with mass as well as flavor conversion. Also shown are the $\pm 1\sigma$ statistical errors for delay with and without mixing in the $1^{st}$, $6^{th}$ and the $11^{th}$ time bins.

**Fig. 4** The variation of $r_1(t)$ with time in SNO. The solid line is for massless neutrinos, the short dashed line for neutrinos with complete flavor conversion but no delay, the long dashed line for neutrinos with only delay and no flavor conversion and the dotted line is for neutrinos with both delay and complete flavor conversion. Also shown are the $\pm 1\sigma$ statistical errors for delay with and without mixing in the $1^{st}$, $8^{th}$ and the $15^{th}$ time bins.

**Fig. 5** The ratio R(t) in SNO for the two cases of fixed and time dependent neutrino temperatures. The solid line and the long dashed line give R(t) for the cases of fixed temperatures and varying temperatures respectively for only delay, while the short dashed line and the dotted line give the corresponding R(t) for delay with mixing. We have also given the $\pm 1\sigma$ statistical errors in the $1^{st}$ and the $4^{th}$ time bin, for the both the curves for fixed and time dependent temperatures.



NC event rate in SNO [$sec^{-1}$]
time [$sec$]
0
20
40
60
80
100
120
140
160
180
200
0
2
4
6
8
10
12
14


Fig. 1

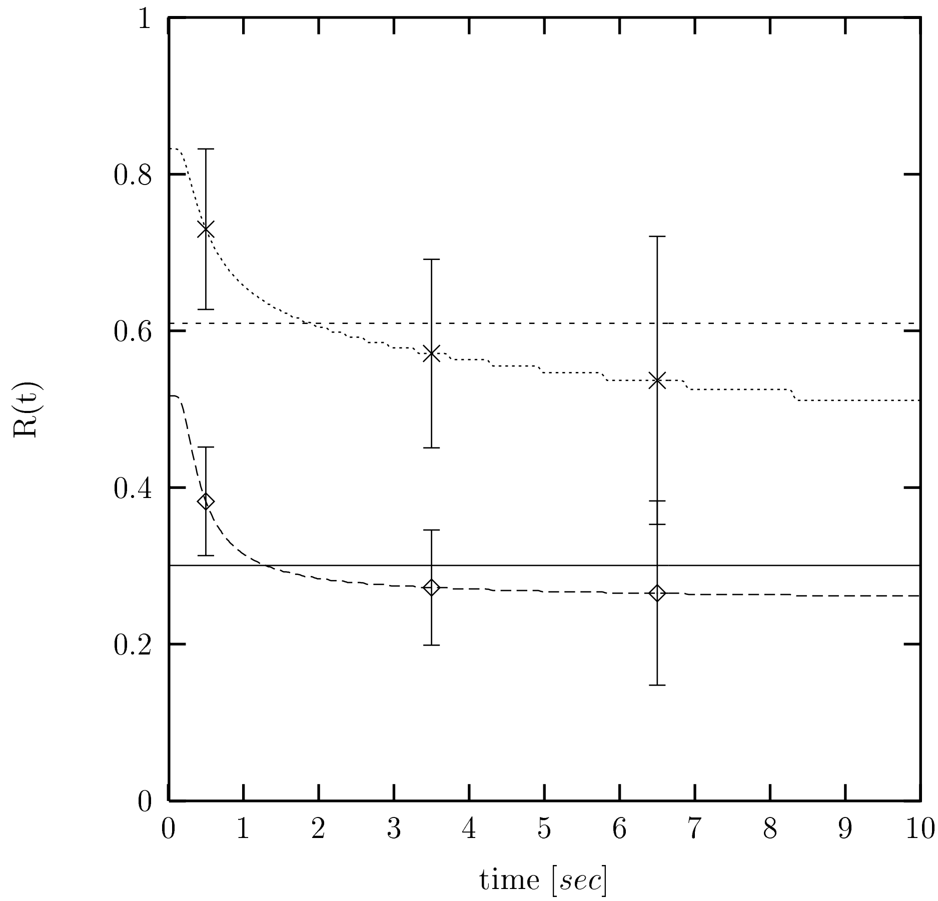



Fig. 2

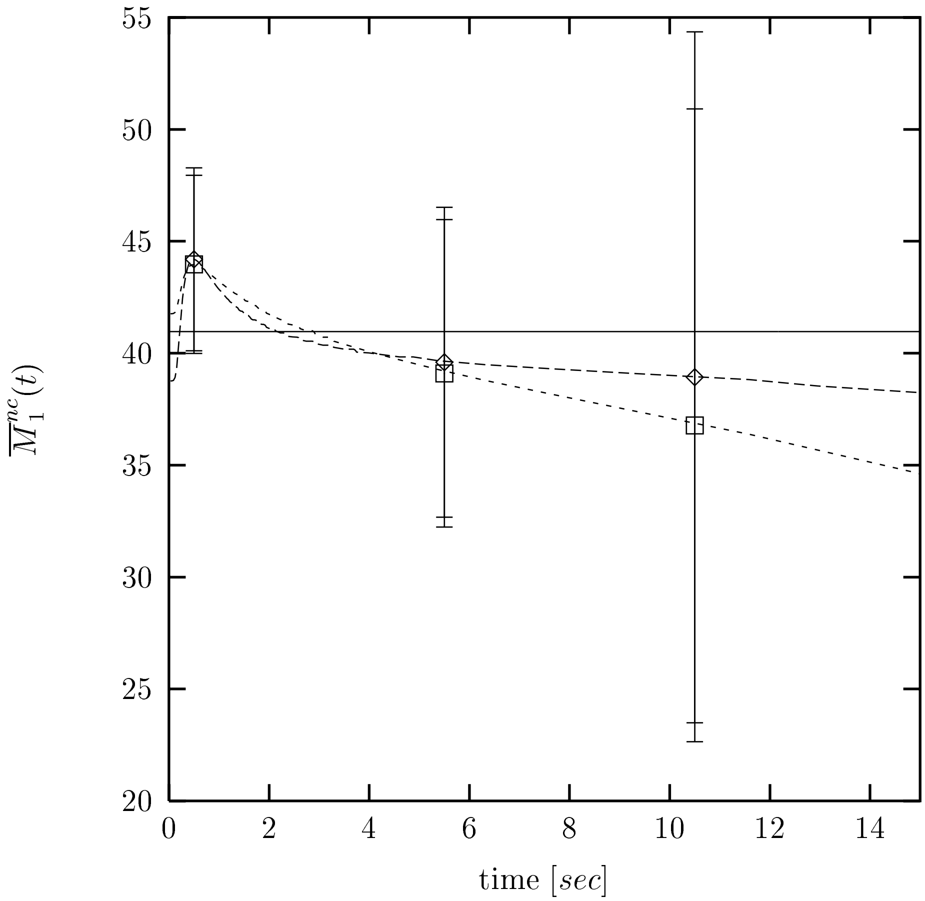



Fig. 3

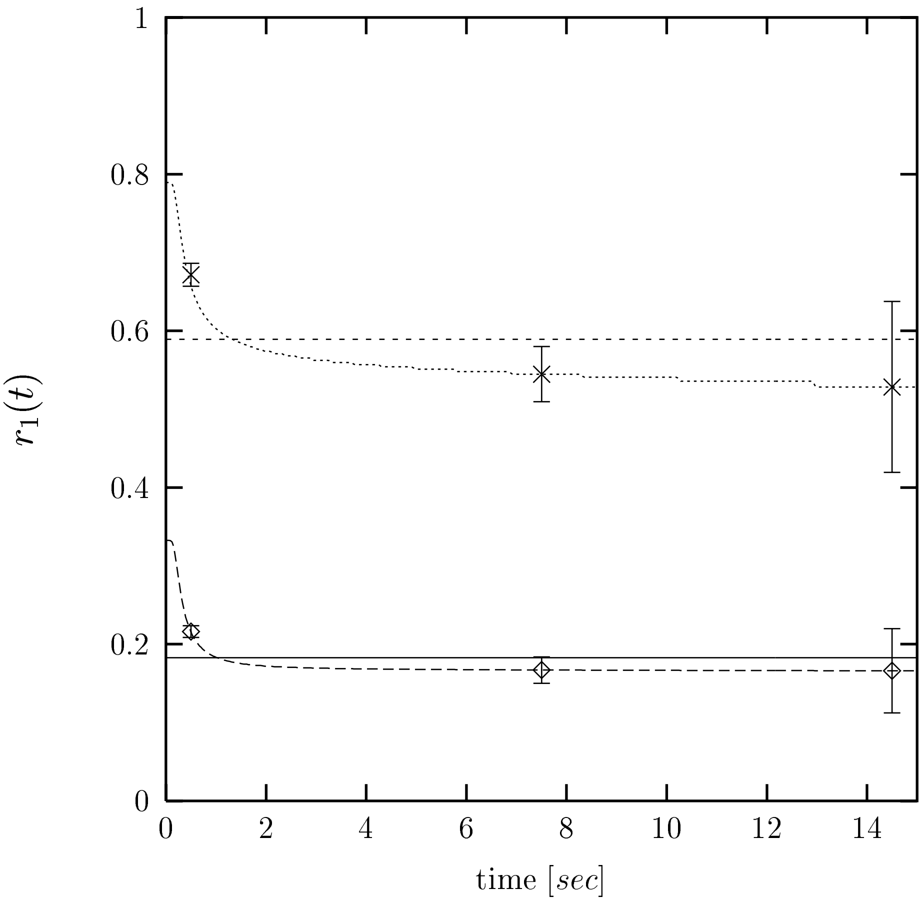



Fig. 4

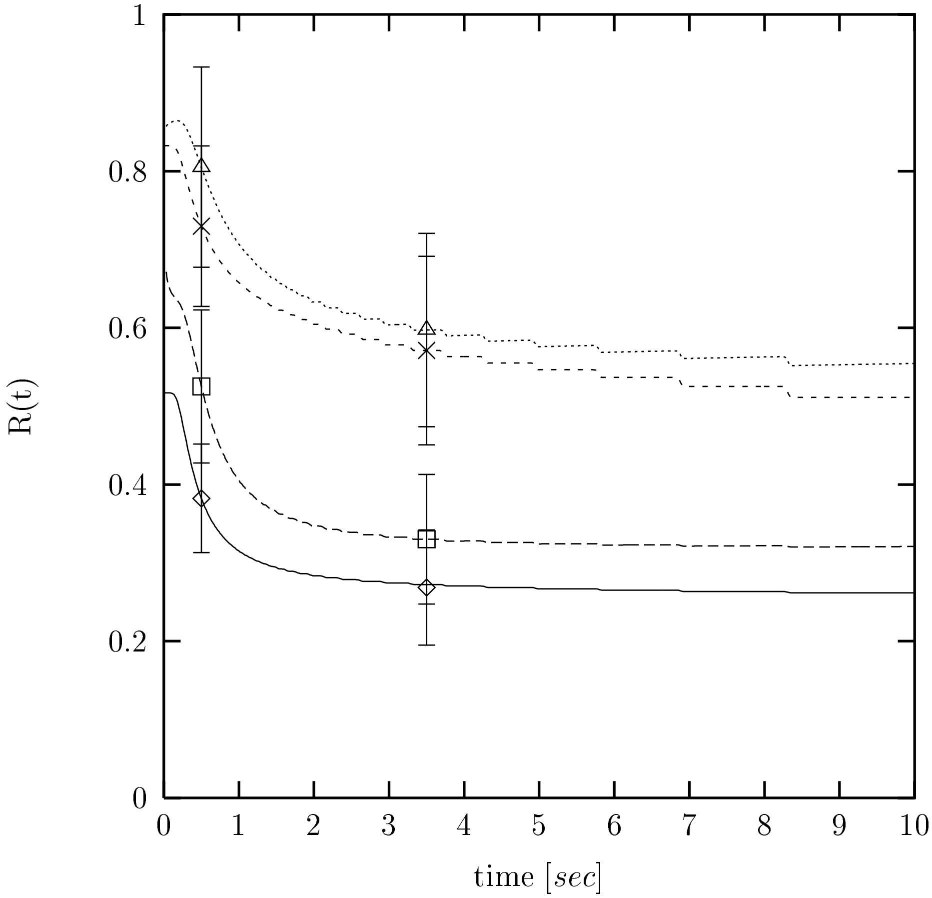



Fig. 5